ANALYTICAL STRUCTURE OF NIGERIAN EURO-BOND UNDER SVENSSON'S MODEL

[1*]Ogungbenle Gbenga Michael and [2]Ogungbenle Simeon Kayode

gbengarising@gmail.com kaykaska@yahoo.com

[1]*Department of Actuarial Science, University of Jos, Jos*

[2]*Department of Finance, Igbinedion University, Okada, Benin City*

ABSTRACT

The term structure is a pointer to market expectations concerning interest rates through expected variations captured by the projected yield curve where the yield curve is a plot of the closing prices of bonds against their corresponding terms to maturity. This paper aims to forecast the term structure of interest rate on Nigerian Eurobond and then estimate the yield curve as a tool for the measurement of economic growth using the Svensson model to allow flexibility in the model to fit observed data. The statistical tool employed for the analysis of data collected are the Ordinary Least Square method, Spearman rho's method and an R^2 adjusted for segmenting the data into four quarters with the aim of generating an aggregate yield curve to prove that the gradient of the yield curve, relative to its time to maturity moves proportionally and has a significant effect on the term structure. The approximating yield curve on Nigeria Euro-bond recommends that Nigerian regulatory authorities should monitor the operations of all financial institutions to hedge against uncertainties in the capital market prices which could discourage investors.

KEY WORDS: Term–structure, Nelson-Siegel model, Yield curve, long term, short term

1 INTRODUCTION

The term structure of interest rate describes the expectations of market operators concerning future volatilities in interest rate and their assessment of fiscal policies. However, the term structure of interest rates and the direction of the yield curve is an instrument that may be used to appraise the aggregate credit market environment. The time structure architecture of interest rates describes the relationship between yield and maturities of the same type of debt instruments with varying risk. Chen (2019) observes that when two securities are similar and homogeneous in all respect it is probable to be sold in the market at varying interest rates. This paper aims to examine the effect of time to maturity on the Nigerian Eurobond and determines how Nigerian Eurobond can be estimated by an in-sample yield curve model to enable us appraise how well the model fits the observed data. The yield curve is an indication of expectations of future interest rates and other market variables whereas, a forward rate curve, consisting of the forward interest rate seems to be a better indication of the future interest than the yield curve since the forward rate curve shows the expected future time trajectories of the market variables but yield curves indicates expected future mean of the variables.

2 LITERATURE REVIEW

2.1 Theoretical Review

Dobson, Sutch & Vanderford (1976) reported that term structure of interest rates characterizes interest rate as a function of maturity time because it is the basis of investigating the portfolio returns on investment after a defined period facilitating easy assessment of rates and yield. The yield curve is observed as a market benchmark for calculating lending and saving rates and consequently, the prices of debt instruments may vary in the same direction that may be responsible for the yield curve to slope upwards. However, a flattening yield curve proves that longer-term rates are dropping when compared with short term rates basically as a result of economic downturn. When short term rates start exceeding long term rate, the yield curve will be inverted indicating that downturn may occur hence, the lenders' capacity to give credit facilities for long term declines as the expectation of future interest rate remains uncertain to them particularly during economic downturn where two consecutive periods of unfavourable growth of the Gross Domestic Product and foreign demand are known making the yield curve to have an inverted shape. Chen (2019) observes that term structure is the relationship between interest rates or bond yields and distinct terms to maturities confirming that if the term structure is plotted, the interest rates becomes the yield curve which plays a pivot role in the economy wholly as it shows the expectations of market operators over future volatilities in interest rates and the appraisal of fiscal conditions.

2.2 EMPIRICAL REVIEWS

Assumptions have been put forward to explain the term structure of interest rate, however Vasicek (1977) suggested an alternative assumption about the functional form of the term structure which could be efficient in a functional operating market. A true definition of the term structure is obtained in a perfect market but in real life situation, there is no such perfect market because of the presence of uncertainties and the reactions of investors to changes in interest rates.

A counter opinion raised against overly strict description of the term structure in the direction of the expectations theory, shows more intensity on the forward rate curve. Svensson (1994) argued that the spot rates and forward rates can be obtained from each other since they have the same information especially where forward rates is used instead of spot rates is a clear alternative means of representing same information in another way. However, if the intention is on the expected future time trajectories of market determinants rather than expected future time average then applying forward rate is advantageous.

Aljinovic, Poklepovic, & Katalinic. (2012) compared both Nelson-Siegel and Svensson using vector Autoregressive and neural networks and it was reported that Nelson-Siegel model resulted in a better approximation considering smaller mean squared error than projection based on Svensson's. Gulteken & Rogalski (1984) argues on duration measurement that the use of bond data is preferred to fitted yield since there are much inconsistencies in the prices and volatility of debt instruments which can be efficiently measured by simple duration.

Gibson (2010) suggested that the continuous time modeling technique of the term structure should be applied to value and hedge default-free bonds and other interest rate derivatives to ascertain a common structure where most continuous time term structure models could be connected and then functionally related. Consequently, this will be a numerical basis by which the model is compared with, in terms of their similarities and differences, contributions and limitations.

Vasicek (1977) and Vasicek & Fong (1982) put forward a different assumption on the structure of the stochastic process driving interest rates characterizing the term structure in an efficiently operating market so that the resultant spot rates have a defined functional form which depends on a few parameters while the spot rate curves generated from this model may not fit well in bond yields and prices since real yield curves display varied shapes than those exhibited by the equilibrium model. Consequently, a simple technique used in approximating the term structure is to hypothesize that bond payments may occur at a discrete domain of defined dates but assumes no relationship between the discount factor corresponding to the dates. The discount factors could be approximated as the coefficients in a regression equation with the bond payments on the given domain where dates are the independent variable and the bond price are the dependent variable. Svensson, (1993a) and Svensson (1993b) emphasized the import of the forward rate in drawing inference from the expected future rates of interest and inflation differentials. The forward rate relates to the expected future changes through the combination of term, inflation and foreign exchange risk premium. The forward interest rate curve could be used to appraise market expectations of the time trajectories of future short term interest rates, fiscal policies, which differentiate market expectations for short, medium and long term securities making it more flexible than the standard yield curve. Most economies' fiscal policy is interpreted using forward rate as reported in Cox, Ingeroll & Ross (1985) and Svensson (1994). Kopca (2010) critically appraised the initial Nelson-Siegel model which examines the modified Diebold-Li (2006) technique and fixing the parameter lambda on pre-defined value.

Chen (1996) formulated a three-factor model of the term structure of interest rates where the future short rate was intended to depend on the current short rate, the short-term mean of the short rate and the current volatility of the short rate. The author assumed a stochastic form for both the short term mean of the short rate and the volatility of the short rate. The underlying assumptions are premised on wider empirical studies in interest rate characteristics which are described in the model. It was reported that interest rates is probable to be pulled back to a definite future long-run level described as mean reversion. Szenczi (2016) reported that high modeling performance of the dynamic Nelson-Siegel model was capable of generating a well-fitting curve for the Hungarian interest rate computed on different maturity domain. The family of interest rate curve factors are then the input for the financial model and multilayer artificial neural network based on feed-forward structure capable of permitting forecast on the term structure of interest rates for about 5-10 years so that, the approximated term structure could constitute a sound basis of trading because of its strength to permit non-linear relationships. Interest rate is a fiscal policy set by regulatory body to control macroeconomic variables such as inflation. It is clear that every regulatory body applies a structured functional model for modeling the term structure but Nigeria has not put in place any defined known model for measurement. This paper concentrates in estimating the term structure of interest rate as it relates to Nigerian Eurobond and assuming that the yield curve would be an effective tool for the measurement of economic growth in the nearest future using the Svensson model (1994).

3 MATERIAL AND METHODS: NELSON-SIEGELMODEL

Nelson-Siegel (1987) assumes that the instantaneous forward rate $g(n)$ is a second-order differential equation with two equal roots

$$g(n,\Sigma_1)=\sigma_0+\sigma_1\exp\left(\frac{-n}{\tau_1}\right)+\sigma_2\frac{n}{\tau_1}\exp\left(\frac{-n}{\tau_1}\right) \tag{1}$$

$\Sigma_1=(\sigma_0,\sigma_1,\sigma_2,\tau_1)$ is a vector of parameters with $\sigma_0>0$ and $\tau_1>0$.The forward rate function has three components: σ_0 is a fixed number, $\sigma_1\exp\left(\frac{-n}{\tau_1}\right)$ is an exponential term which is monotonically decreasing or increasing if σ_1 is negative towards zero as a function of the time to maturity and $\sigma_2\frac{n}{\tau_1}\exp\left(\frac{-n}{\tau_1}\right)$ is the term generating a hump-shape as a function of time to maturity. But when the time to maturity or settlement approaches infinity, the forward rate approaches the constant σ_0 and when the time to maturity approaches zero, the forward rate approaches the constant $\sigma_0+\sigma_1$. By taking the average of the forward rate function from zero to maturity, we obtain the following expression for the spot rate

$$r(n,\Sigma_1)=\frac{1}{n}\int_0^n\left[\sigma_0+\sigma_1\exp\left(\frac{-s}{\tau}\right)+\sigma_2\left(\frac{s}{\tau}\right).\exp\left(\frac{-s}{\tau}\right)\right]ds \tag{2}$$

$r(\theta,n)$ is the continuously compounded spot rate for a zero coupon traded at time θ, the trade date , that matures at time $\theta<n$

Making the following change of variables in equation (2): $s=z\tau, ds=\tau dz$ (3)

We get,

$$r(n,\Sigma_1)=\sigma_0+\left(\sigma_1\frac{\tau}{n}\right)\int_0^{\frac{n}{\tau}}e^{-z}dx+\left(\sigma_2\frac{\tau}{n}\right)\int_0^{\frac{n}{\tau}}ze^{-z}dz \tag{4}$$

$$r(n,\Sigma_1)=\sigma_0+\left(\sigma_1\frac{\tau}{n}\right)\left[-e^{-z}\right]_0^{\frac{n}{\tau}}+\left(\sigma_2\frac{\tau}{n}\right)\left\{\left[-ze^{-z}\right]_0^{\frac{n}{\tau}}-\int_0^{\frac{n}{\tau}}e^{-z}dz\right\} \tag{5}$$

$$r(n,\Sigma_1)=\sigma_0+\sigma_1\left[\frac{1-e^{\frac{-n}{\tau}}}{\frac{n}{\tau}}\right]+\sigma_2\left[\frac{1-e^{\frac{-n}{\tau}}}{\frac{n}{\tau}}-e^{\frac{-n}{\tau}}\right] \tag{6}$$

Finally, we get the following formula for spot rates

$$r(n,\Sigma_1)=\sigma_0+\sigma_1\left[\frac{1-\exp\left(\frac{-n}{\tau}\right)}{\frac{n}{\tau}}\right]+\sigma_2\left[\frac{1-\exp\left(\frac{-n}{\tau}\right)}{\frac{n}{\tau}}-\exp\left(\frac{-n}{\tau}\right)\right] \tag{7}$$

If the forward rate $g(\theta,\omega)$ commences at time θ with maturity ω, and we divide the interval $[\theta,\omega]$ so that $n\to\infty$,then $\delta\theta\to 0$, we can re-write the forward rate function as

$$g(\theta,\omega)=\frac{1}{\omega-\theta}\int_{\theta}^{\omega}g(\tau,s)ds \tag{8}$$

Where $g(\tau,s)$ is the instantaneous forward rate and by taking the average of the equation, we have;

$$g(\theta,\omega)=\frac{1}{\omega-\theta}\int_{\theta}^{\omega}\sigma_0+\sigma_1\exp\left(\frac{-s}{\tau}\right)+\sigma_2\left[\left(\frac{s}{\tau}\right)\exp\left(\frac{-s}{\tau}\right)\right]ds$$

$$=\sigma_0+\sigma_1\frac{\tau}{\omega-\theta}\left[\exp\left(\frac{-\theta}{\tau}\right)-\exp\left(\frac{-\omega}{\tau}\right)\right]+\sigma_2\frac{\tau}{\omega-\theta}\left[\exp\left(\frac{-\theta}{\tau}\right)-\exp\left(\frac{-\omega}{\tau}\right)-\frac{\omega}{\tau}\exp\left(\frac{-\omega}{\tau}\right)+\frac{\theta}{\tau}\exp\left(\frac{-\theta}{\tau}\right)\right]$$ where the coefficient of the model for the observation of the flexibility is (9)

$$\lim_{n\to\infty}r(n,\Sigma_1)=\sigma_0 \tag{10}$$

So that σ_0 represents the contribution of the long term component of the model

$$\lim_{n\to\infty}r(n,\Sigma_1)=\sigma_0+\sigma_1 \tag{11}$$

then $(\sigma_0+\sigma_1)$ represents the instantaneous interest rate. (12)

4 FIXING THE PARAMETERS OF THE SVENSSON MODEL

The extended Nelson-Siegel-model is meant to capture the various shapes of spot rates with an intent to prepare a smooth term structure in both zero rates and forward rates and to furnish flexibility in the model to fit the observed data designed to generate a sound estimation to value instruments that have cash-flows on other days than the market data by adding a second hump-shape $\sigma_3\frac{n}{\tau_2}\exp\left(\frac{-n}{\tau_2}\right)$ with two additional parameters $\sigma_3>0$ and $\tau_2>0$. Then the forward rate function can be;

$$g(n,\Sigma_2)=\sigma_0+\sigma_1\exp\left(\frac{-n}{\tau_1}\right)+\sigma_2\frac{n}{\tau_1}\exp\left(\frac{-n}{\tau_1}\right)+\sigma_3\frac{m}{\tau_2}\exp\left(\frac{-n}{\tau_2}\right) \tag{13}$$

Where $\Sigma_2=(\sigma_0,\sigma_1,\sigma_2,\sigma_3,\tau_1,\tau_2)$ are the parameters we want to approximate. It is important to note that Nelson and Siegel model does not include the term, σ_3 but Svensson's extension allows an additional turning point in the estimated curve. The spot rate can then be obtained by integrating the forward rate in the interval $\tau\le r\le\omega$ (14)

$$r(\theta,\omega)=\left(\frac{\int_{\tau=1}^{\omega}g(\theta,\tau)dr}{\omega-\theta}\right) \tag{15}$$

let $r(u)$ denote the spot rate $r(\theta,\theta+n)$ of time to maturity n for a given trade date θ, then, we have

$$r(n,\Sigma_2)=\sigma_0+\sigma_1\left[\frac{1-\exp\left(\frac{-n}{\tau_1}\right)}{\frac{n}{\tau_1}}\right]+\sigma_2\left[\frac{1-\exp\left(\frac{-n}{\tau_1}\right)}{\frac{n}{\tau_1}}-\exp\left(\frac{-n}{\tau_1}\right)\right]+\sigma_3\left[\frac{1-\exp\left(\frac{-n}{\tau_2}\right)}{\frac{n}{\tau_2}}-\exp\left(\frac{-n}{\tau_2}\right)\right]$$

This can be re-arranged to obtain

$$\mathrm{r_s}(\lambda,\tau)=\sigma_{0\theta}+\sigma_{1\theta}\left[\frac{1-\mathrm{e}^{-\lambda_{1\theta}\tau_1}}{\lambda_{1\theta}\tau_1}\right]+\sigma_{2\theta}\left[\frac{1-\mathrm{e}^{-\lambda_{1\theta}\tau_1}}{\lambda_{1\theta}\tau_1}-\mathrm{e}^{-\lambda_{1\theta}\tau_1}\right]+\sigma_{3\theta}\left[\frac{1-\mathrm{e}^{-\lambda_{2\theta}\tau_2}}{\lambda_{2\theta}\tau_2}-\mathrm{e}^{-\lambda_{2\theta}\tau_2}\right] \quad (16)$$

$$\Sigma_2=(\sigma_0,\sigma_1,\sigma_2,\sigma_3,\tau_1,\tau_2)$$

This could be approximated for every trade date by minimizing yield errors. The discount function is applied in predicting bond prices and simultaneously employed in minimizing the sum squared yield errors between estimated yields and observed yields. The discount function d is given by,

$$\lambda r(\lambda,\Sigma_2)=-\tau\log_e\left(d(\lambda,b)\right) \quad (17)$$

5 METHOD OF DATA ANALYSIS

The statistical tools employed for the study are SPSS 23 and ad-in excel environment which are software packages purposely programmed for analyzing data. The data presented in this study involves the daily closing of the Nigerian Eurobond yield comprising data for the period January to December 2018 to analyze and fit the Nigerian Eurobond yield curve using the Svensson model.

The data for the twelve months were analyzed on quarterly bases for the years 2018 and the result and findings were displayed in graphical and tabular forms to include other statistics not captured on the curve.

Table 1. First quarter Descriptive Statistics

Tenor (τ)	N	Minimum	Maximum	Mean	Std. Deviation	Variance

	Statistic	Statistic	Statistic	Statistic	Std. Error	Statistic	Statistic
2 years	44	4.1350	4.8910	4.530432	.0415433	.2755673	.076
4 years	44	4.0140	5.1160	4.827205	.0381422	.2530065	.064
5 years	44	4.8250	6.5880	5.230273	.0590231	.3915147	.153
9 years	44	4.5970	6.6450	6.098409	.0716375	.4751897	.226
14 years	44	5.9210	7.1790	6.711114	.0561927	.3727401	.139
29 years	44	6.8470	7.6730	7.252773	.0412490	.2736152	.075

Authors' computation via SPSS 23

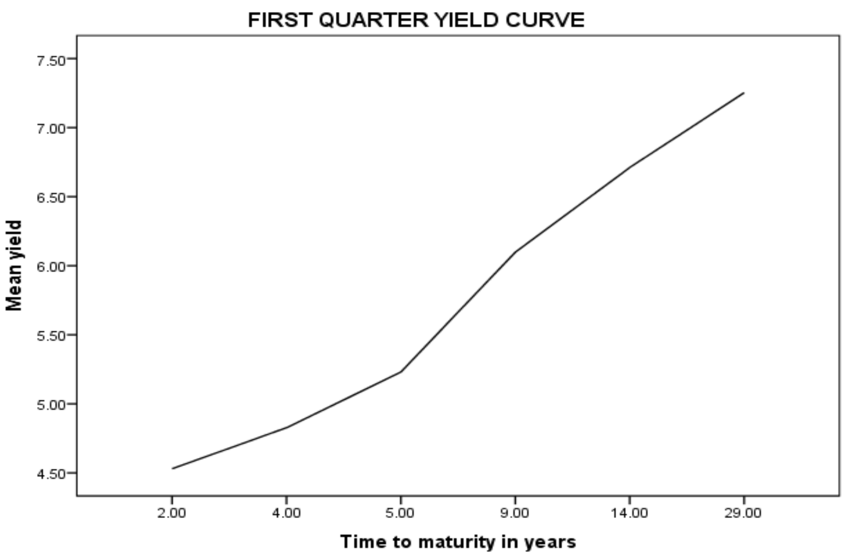

FIG.1 AUTHORS' COMPUTATION VIA SPSS 23

The descriptive analysis of the first quarter as shown in table 1 above goes with time to maturity of 2 years, 4 years, 5 years, 9 years, 14 years and 29 years and a mean yield of 4.530432, 4.827205, 5.230273, 6.098409, 6.711114 and 7.252773 respectively. The variance of the data collected is relatively low with high variation on the 9 years time to maturity and lower at the 4 years maturity. The yield curve depicted above in figure 2 shows that the yield curve is moving upward as time to maturity increases.

Table 2. Second quarter Descriptive Statistics

	N	Minim	Maxim	Mean		Std. Deviation	Variance
Tenor (τ)	Statistic	Statistic	Statistic	Statistic	Std. Error	Statistic	Statistic
2 years	45	4.4620	6.2660	5.075422	.0754607	.5062059	.256

4 years	45	4.9100	6.3560	5.352178	.0581200	.3898810	.152
5 years	45	5.0870	6.8500	5.661556	.0700695	.4700406	.221
9 years	45	5.5110	7.6750	6.628289	.0679810	.4560301	.208
12 years	45	6.4940	8.1100	7.089244	.0638245	.4281474	.183
14 years	45	6.6120	8.3260	7.238578	.0663314	.4449644	.198
20 years	45	7.05	8.52	7.5962	.05890	.39511	.156
29 years	45	7.16	8.65	7.6980	.05939	.39842	.159

Authors' computation via SPSS 23

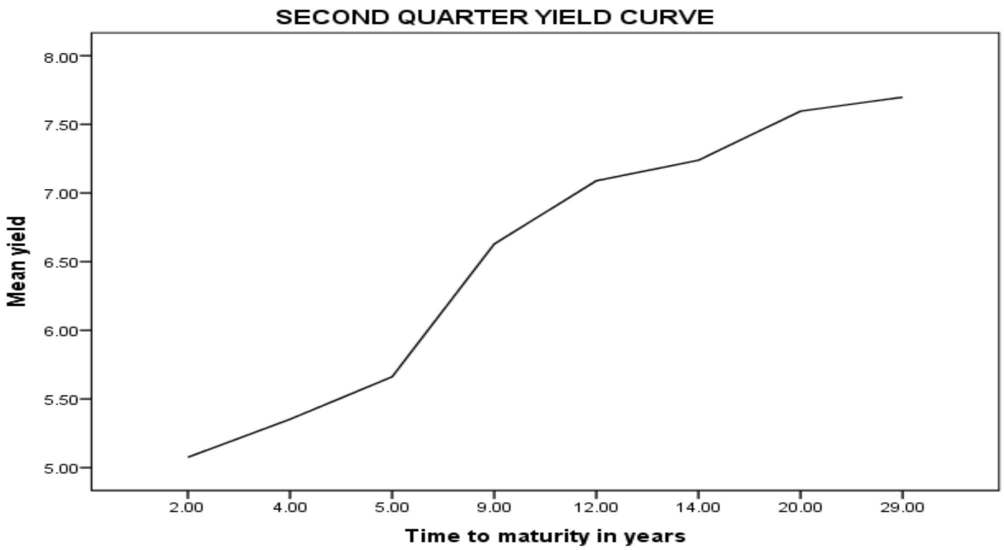

FIG.2 AUTHORS' COMPUTATION VIA SPSS 23

The number of time to maturity of the Nigerian Eurobond in the second quarter increases with the addition of 12 years maturity and 20 years time to maturity. The overall time to maturities of the quarter include 2 years, 4 years, 5 years, 9 years, 12 years, 14 years, 10 years and 29 years with the corresponding mean yield of 5.075422, 5.352178, 5.661556, 6.628289, 7.238578, 7.5962, and 7.6980. The variance of this quarter is higher compared to the first quarter with 2 years' time to maturity having the highest variance while the 4 years time to maturity have the lowest variance. The yield curve shown in figure 3 reveals that the curve is also moving upward as it is in the previous quarter.

Table 3. Third quarter descriptive statistics

	N	Minimum	Maximum	Mean		Std. Deviation	Variance
Tenor (τ)	Statistic	Statistic	Statistic	Statistic	Std. Error	Statistic	Statistic

2 years	44	4.6840	5.8460	5.178023	.0456638	.3028993	.092
4 years	44	5.2820	6.1710	5.724455	.0387018	.2567184	.066
5 years	44	5.7710	6.6370	6.190932	.0390991	.2593541	.067
9 years	44	6.7890	7.7790	7.283818	.0432829	.2871063	.082
12 years	44	7.1270	8.1420	7.629477	.0426754	.2830768	.080
14 years	44	7.3980	8.4110	7.867136	.0420926	.2792106	.078
20 years	44	7.71	8.71	8.1622	.04187	.27776	.077
29 years	44	7.83	8.80	8.2826	.03862	.25616	.066

Authors' computation via SPSS 23

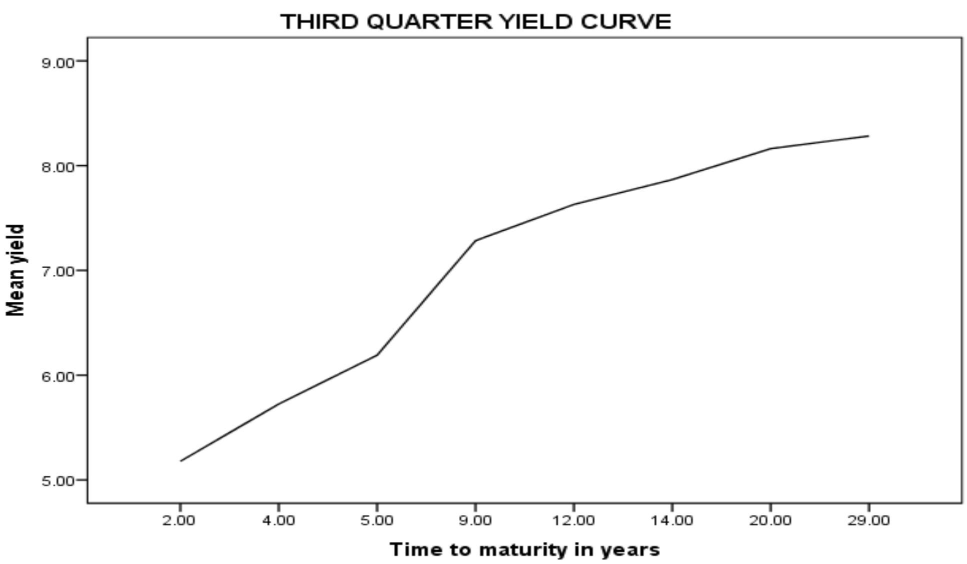

FIG.3 AUTHORS' COMPUTATION VIA SPSS 23

In the third quarter descriptive statistics shown in table 3, the time to maturities available are 2 years, 4 years, 5 years, 9 years, 12 years, 14 years, 10 years and 29 years with respective mean yield of 5.178023, 5.724455, 6.190932, 7.283818, 7.629477, 7.867136, 8.1622 and 8.2826. The quarter experienced a higher yield when compared to the previous quarters. The quarter also experiences an increase in variance more than the previous months. The third quarter yield curve as depicted in figure 3 above indicated an upward moving slope just as in the previous quarters.

Table 4. Fourth quarter Descriptive Statistics

	N	Minimu m	Maximum	Mean		Std. Deviation	Variance
Tenor (τ)	Statistic	Statistic	Statistic	Statistic	Std. Error	Statistic	Statistic

2 years	42	4.8510	6.0730	5.435524	.0661428	.4286541	.184
4 years	42	5.5100	6.5850	6.043381	.0509444	.3301575	.109
5 years	42	5.9790	7.4720	6.687786	.0788105	.5107502	.261
9 years	42	7.0450	8.5820	7.834857	.0702595	.4553336	.207
12 years	42	7.4300	8.8420	8.257643	.0643830	.4172497	.174
14 years	42	7.5480	9.1700	8.469810	.0778415	.5044706	.254
20 years	42	7.90	9.15	8.6805	.05226	.33868	.115
29 years	42	7.99	9.15	8.7056	.04707	.30502	.093

Authors' computation via SPSS 23

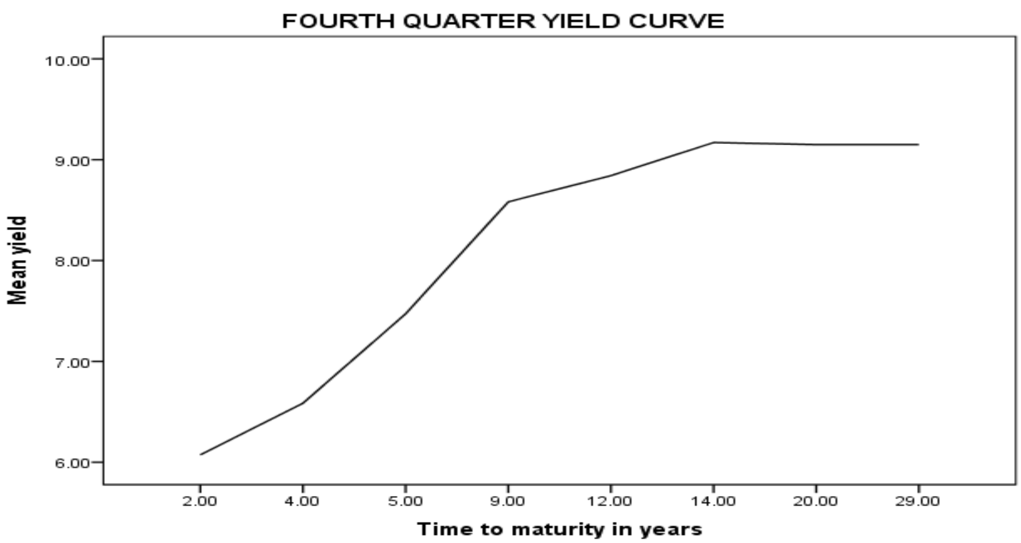

FIG.4 AUTHORS' COMPUTATION VIA SPSS 23

In the fourth quarter descriptive statistics as shown in table 4, the time to maturities available are 2 years, 4 years, 5 years, 9 years, 12 years, 14 years, 10 years and 29 years with respective mean yield of 5.435524, 6.043381, 6.687786, 7.834857, 8.257643, 8.469810, 8.6805 and 8.7056. The fourth quarter experiences a tremendous increase in yield. The quarter has a maximum yield of 9.1700, 9.15, and 9.15, of 14 years, 20 years and 29 years' time to maturity respectively. The quarter experiences higher level of variation with the highest in the 5 years maturity and the lowest at 29 years. The yield curve of the fourth quarter as depicted in figure 4 above shows that the yield curve is sloping upward giving an increase of time to maturity.

Table 5. Aggregate Descriptive Statistics

	N	Minimum	Maximum	Mean		Std. Deviation	Variance
TENOR (τ)	Statistic	Statistic	Statistic	Statistic	Std. Error	Statistic	Statistic

2 YEARS	175	4.14	6.27	5.0506	.03840	.50792	.258
4 YEARS	175	4.01	6.59	5.4797	.04140	.54767	.300
5 YEARS	175	4.83	7.47	5.9325	.05183	.68565	.470
9 YEARS	175	4.60	8.58	6.9495	.05879	.77778	.605
12YEARS	145	6.49	8.84	7.5734	.05150	.62018	.385
14 YEARS	175	5.92	9.17	7.5595	.05837	.77216	.596
20 YEARS	145	7.05	9.15	8.0615	.04776	.57511	.331
29 YEARS	175	6.85	9.15	7.9749	.04789	.63348	.401

Authors' computation via SPSS 23

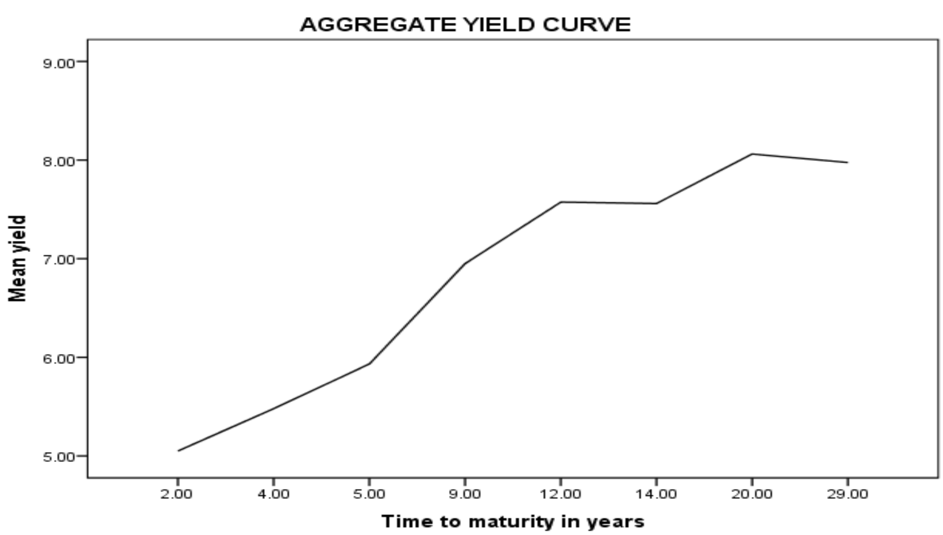

FIG.5 AUTHORS' COMPUTATION VIA SPSS 23

The aggregate descriptive statistics as shown above in table 5, the time to maturities available are 2 years, 4 years, 5 years, 9 years, 12 years, 14 years, 10 years and 29 years with respective mean yield of 5.0506, 5.4797, 5.9325, 6.9495, 7.5734, 7.5595, 8.0615 and 7.9749. The overall statistics experience a higher level of variance. These can be attributed to the numerous data and the monthly market behavior. The aggregate yield curve of as depicted in figure 5 above shows that the yield curve is sloping upward giving an increase of time to maturity, but a decline was experienced on the last tenor.

6 ESTIMATION OF SVENSSON MODEL PARAMETERS

We adopted the lambda values 0.089 and 0.6559 for $\lambda_{\theta 1}$ and $\lambda_{\theta 2}$ respectively computed in Poklepovic, Aljinovic, and Marasovic, (2014) since our $\lambda_1$and λ_2 values are very close to theirs. The results obtained are presented in the table below:

$$Table\ 6. Model\ coefficient$$

First quarter	Unstandardized Coefficients σ	Std. Error	Standardized Coe. Beta	T	Sig.
σ_0	6.763	.371		18.213	.003
σ_1	-.917	.656	-.173	-1.397	.297
σ_2	3.990	1.007	.305	3.960	.058
σ_3	-5.947	1.735	-.537	-3.427	.076
Second quarter	**Unstandardized Coefficients σ**	**Std. Error**	**Standardized Coe. Beta**	**T**	**Sig.**
σ_0	7.068	.114		62.089	.000
σ_1	-.010	.195	-.002	-.049	.963
σ_2	3.744	.320	.292	11.694	.000
σ_3	-7.935	.541	-.716	-14.671	.000
Third quarter	**Unstandardized Coefficients σ**	**Std. Error**	**Standardized Coe. Beta**	**T**	**Sig.**
σ_0	6.324	.428		14.777	.000
σ_1	-.715	.735	-.116	-.973	.385
σ_2	8.468	1.204	.580	7.036	.002
σ_3	-4.004	2.033	-.318	-1.969	.120
Fourth quarter	**Unstandardized Coefficients σ**	**Std. Error**	**Standardized Coe. Beta**	**T**	**Sig.**
σ_0	5.516	.635		8.683	.001
σ_1	.306	1.091	.046	.281	.793
σ_2	11.539	1.787	.730	6.459	.003
σ_3	-4.392	3.018	-.322	-1.455	.219
Aggregate	**Unstandardized Coefficients σ**	**Std. Error**	**Standardized Coe. Beta**	**T**	**Sig.**
σ_0	6.172	.566		10.910	.000
σ_1	.262	.971	.043	.270	.800
σ_2	7.402	1.591	.510	4.653	.010
σ_3	-6.797	2.688	-.542	-2.529	.065

Authors' computation via SPSS 23

RESEARCH QUESTIONS

6.1 Does time to maturity has an effect on term structure of interest rate?

The relationship that existed between time to maturity of a set of similar securities and its corresponding yield is referred to as the term structure of interest rate. To determine whether time to maturity has an effect on the term structure of interest rate, we use the aggregate yield curve as shown below.

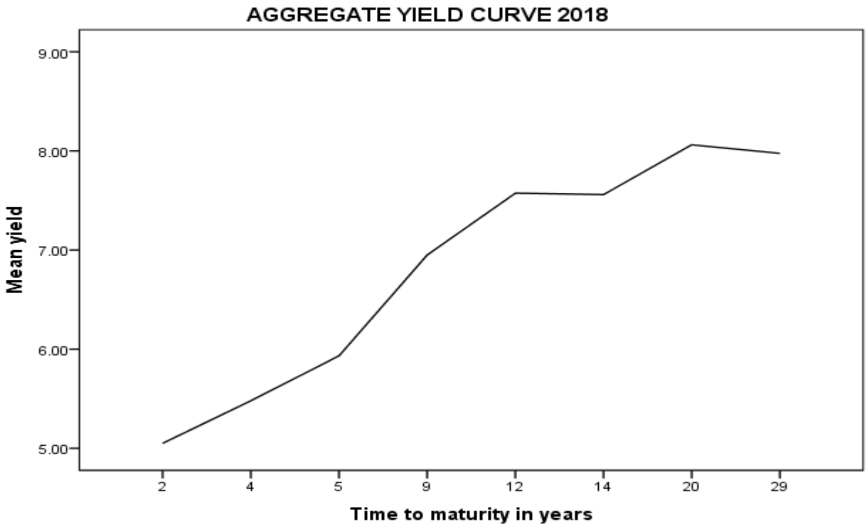

FIG 6. AUTHORS' COMPUTATION VIA SPSS 23

Table 7. Correlations

		Yield	Time to maturity
Spearman's rho yield	Correlation coefficient Sig. (2-tailed) N	1.000 . 8	952** .000 8
Time to maturity	Correlation coefficient Sig.(2-ailed) N	952** .000 8	1.000 . 8

Author's computation via SPSS 23

The curve shown above in figure 5 indicates that the slope of the yield is proportional to its time to maturity. That is, when time to maturity becomes longer in duration, the yield also becomes higher in percentage. We can conclude that time to maturity has a significant effect on term structure of interest rate. Also, the evidence shown in table 7 of the spearman rho's correlation reveals that time to maturity has an effect on term structure of interest rate.

6.2 How well does the model fit in the observed data?

The fitness of the model can be determined by R square and R square adjusted. The results for the four quarters and aggregate R square are presented in the table below.

Table 8. Model summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate

First quarter	.999[a]	.999	.997	.0613841
Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
Second quarter	1.000a	1.000	1.000	.0197945
Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
Third quarter	.999a	.998	.996	.0744169
Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
Fourth quarter	.998a	.996	.993	.1104579
Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
Aggregate	.998a	.996	.993	.0983607

Authors' computation via SPSS 23

From the findings in the Model Summary, the adjusted R square is the coefficient of multiple determinations which is the variance percentage in the dependent variable as explained by the independent variable. From the above table, the R square adjusted for the first quarter, second quarter, third quarter, fourth quarter and the aggregate data are 0.997, 1.000, 0.996, 0.993 and 0.993 This shows that the observed data can be explained by the model with 99.7%, 100%, 99.6%, 99.3% and 99.3% degree of accuracy for the first quarter, second quarter, third quarter, fourth quarter and the Aggregate respectively. We can conclude that the Svensson model fit in very well with the observed data given the estimated parameters.

6.3 How can the Eurobond be estimated by an in-sample yield?

Having established the fitness of the model, the model can be used to estimate other time to maturity not included in the observed data. Since the parameters have been estimated, we substitute the parameters value into the model as below.

Aggregate model

$$r_s(\lambda,\tau) = 6.172 + 0.262\left[\frac{1-e^{-0.089\tau_1}}{0.089\tau_1}\right] + 7.402\left[\frac{1-e^{-0.089\tau_1}}{0.089\tau_1} - e^{-0.089\tau_1}\right] - 6.797\left[\frac{1-e^{-0.6559\tau_2}}{0.6559\tau_2} - e^{-0.6559\tau_2}\right]$$

First quarter model

$$r_s(\lambda,\tau) = 6.763 - 0.917\left[\frac{1-e^{-0.089\tau_1}}{0.089\tau_1}\right] + 3.99\left[\frac{1-e^{-0.089\tau_1}}{0.089\tau_1} - e^{-0.089\tau_1}\right] - 5.947\left[\frac{1-e^{-0.6559\tau_2}}{0.6559\tau_2} - e^{-0.6559\tau_2}\right]$$

Second quarter

$$r_s(\lambda,\tau) = 7.068 - 0.01\left[\frac{1-e^{-0.089\tau_1}}{0.089\tau_1}\right] + 3.744\left[\frac{1-e^{-0.089\tau_1}}{0.089\tau_1} - e^{-0.089\tau_1}\right] - 7.935\left[\frac{1-e^{-0.6559\tau_2}}{0.6559\tau_2} - e^{-0.6559\tau_2}\right]$$

Third quarter

$$r_s(\lambda,\tau) = 6.324 - 0.715\left[\frac{1-e^{-0.089\tau_1}}{0.089\tau_1}\right] + 8.468\left[\frac{1-e^{-0.089\tau_1}}{0.089\tau_1} - e^{-0.089\tau_1}\right] - 4.004\left[\frac{1-e^{-0.6559\tau_2}}{0.6559\tau_2} - e^{-0.6559\tau_2}\right]$$

Fourth quarter

$$r_s(\lambda,\tau) = 5.516 + 0.306\left[\frac{1-e^{-0.089\tau_1}}{0.089\tau_1}\right] + 11.539\left[\frac{1-e^{-0.089\tau_1}}{0.089\tau_1} - e^{-0.089\tau_2}\right] - 4.392\left[\frac{1-e^{-0.6559\tau_2}}{0.6559\tau_2} - e^{-0.6559\tau_2}\right]$$

The yield curve that reveals the predicted value and the observed value of yield is depicted in figure 7 below.

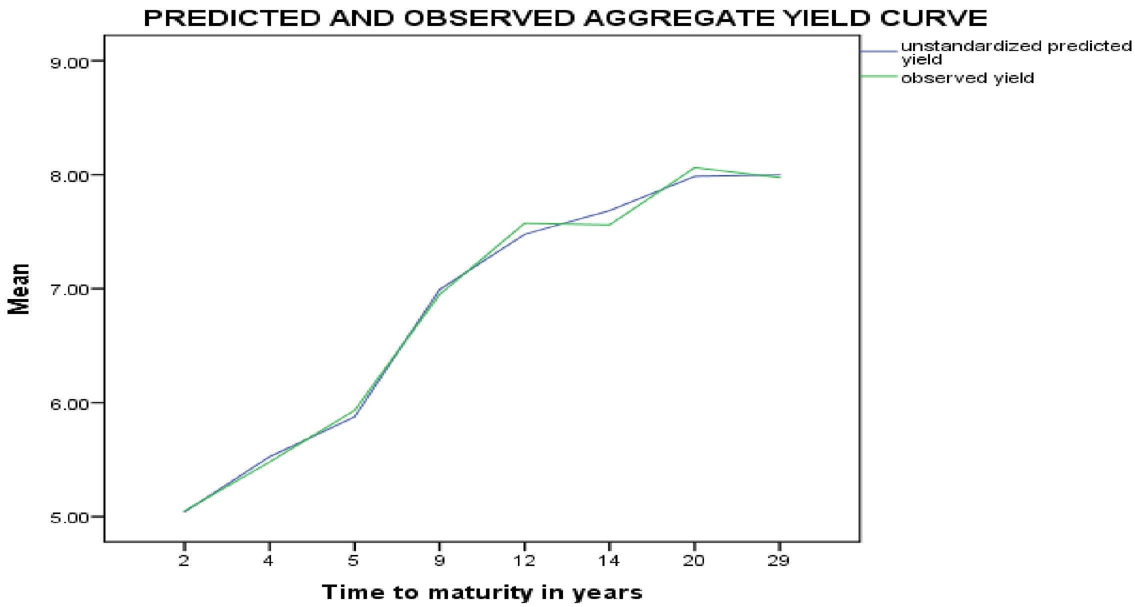

FIG. 7 AUTHORS' COMPUTATION VIA SPSS 23

7 DISCUSSION OF RESULTS

In the descriptive analysis of the four quarters and the aggregate descriptive analysis, it is evident that the quarter with a higher volatility has a higher yield in comparison with others. This shows that the higher the risk of a security, the higher the return. From the yield curves, we find out that the yield curves are both sloping upward, implying that, the higher the time to maturity, the higher the yield. From the positive slope of the Nigerian Eurobond yield curve for the period 2018 of this study, we can infer that the returns on investment on Nigerian Eurobond is anticipated to rise as time to maturity rises. The yield curve positive slope reflects the market expectation that in the future, the higher yield on long-term maturities should adequately reward market operators over the risk they have taken.

The result from the Svensson model analysis using ordinary least square method shows a high level of goodness of fit, the R square adjusted for the first quarter, second quarter, third quarter, fourth quarter and the Aggregate data are 0.997, 1.000, 0.996, 0.993 and 0.993 respectively. This shows that the observed data can be explained by the model with 99.7%, 100%, 99.6%, 99.3% and 99.3% degree of accuracy for the first quarter, second quarter, third quarter, fourth quarter and the Aggregate model respectively.

We also find out that time to maturity has a significant effect on the term structure of interest rate. The correlation coefficient of 0.95of the spearman rho in table 7 reveals the above fact.

8 CONCLUSION AND RECOMMENDATIONS

The data for the study was segmented into four quarters with the aim of getting an aggregate yield curve for the Nigerian Eurobond over period January to December, 2018 stating that the quarter with a higher volatility has a higher yield. That is, the higher the risk of the debt instrument, the higher the expected return. Primarily, the yield curve could be applied to predict the future of interest rate as proposed by expectation theory.

The time to maturity has a significant effect on the term structure with a correlation coefficient of 0.95 indicating that the slope of the yield is proportional to its time to maturity because as time to maturity becomes longer in duration, the yield also becomes higher in percentage as seen in table 6. A good indication of the term structure of interest rate is the forward rate curve which is responsible for the forecast of expected future time trajectories of future short term rates and fiscal policies which is of valuable significance to the analysis of market expert. The second-hump shape in Svensson (1993a) would allow a smooth term structure in both zero rates and forward rates so as to increase flexibility in the model and to fit the observed data to have a better projection of cash-flow values. Based on our results, we will suggest the following policy recommendations. (1), other econometric based models should be tested on data to analyze the daily yields to have an accurate data for the approximation of the term structure for future predictions. (2), Regulatory bodies are required to monitor the operations of financial institutions to minimize fluctuation of stock prices which could discourage investors from investing in a particular sector of the economy. (3), The Svensson model could only be applied when it becomes imperative to increase flexibility in the model to fit the data that could be used for regulatory decision making.

REFERENCES

Aljinovic, Z., Poklepovic, T., and Katalinic K. (2012): ''Best fit model for yield curve estimation''. Croatian Operational Research Review 3, 28-39.

Chen B (2019).*Shapes and Transitions of the Interest Rate Term Structure*, A doctoral thesis department of Economics, The City University of New York.

Chen L. (1996), Chen L. (1996), A Three-Factor Model of the Term Structure of Interest Rates.: Interest Rate Dynamics, Derivatives Pricing, and Risk Management. Lecture Notes in Economics and Mathematical Systems, 435, 1-36 Springer, Berlin, Heidelberg

Cox, J.C., Ingeroll, J.E. and Ross, S.A., (1985): ''The theory of the term structure of interest rate''. *Econometrics* 53, 385-407.

Diebold, F. X. and Li, C. (2006): "Forecasting the Term Structure of Government Bond Yields", Journal of Econometrics, 130(2), 337–364.

Dobson S. W, Sutch R, and Vanderford D. E (1976), ''An Evaluation of Alternative Empirical Models of the Term Structure of Interest Rates''. *The Journal of Finance* 31(4), 1035-65

Gibson R, Lhabitant F.S and Talay. D (2010), "Modeling the Term Structure of Interest Rates", A Review of the Literature. *Foundations and Trends(R) in Finance*, 5(1–2), 1-156

Gultekin N and Rogalski R (1984), ''Alternative Duration specifications and the measure empirical tests'', *Journal of business 57, 241-264*

Kopca M. Bc (2010), Prediction of the yield curve Using a no-arbitrage Nelson-Siegel model. A masters thesis, Department of applied mathematics and statistics, Comenius University in Bratislava

Nelson, C.R. and Siegel, A.F. (1987): "Parsimonious modeling of yield curve". *Journal of Business*60, 473-489.

Poklepovic, T., Aljinovic, Z. and Marasovic, B. (2014): ''The Ability of Forecasting The Term Structure of Interest Rates Based on Nelson-Siegel and Svensson Model''. *International Journal of Economics and Management Engineering* 8(3), 718-724

Svensson, L.E.O. (1993a): ''The Forward Interest Rate Curve –An indicator of market expectations of future interest rate, inflation and exchange rates''. *Swedish Economics Debatt* 21-3, 219-234.

Svensson, L.E.O. (1993b): Monetary policy with flexible exchange rates and forward as indicators. *IIES Seminar Paper* No. 559.

Svensson, L. E. (1994): "Estimating and Interpreting Forward Interest Rates: Sweden 1992-1994", Centre for Economic Policy Research, Discussion Paper 1051.

Szenczi, R (2016): Forecasting the Hungarian term structure of interest rates with econometrics and neural network methods. A Masters thesis, Department of Finance, Corvinus University of Budapest.

Vasicek, O.A. (1977): ''An equilibrium characterization of the term structure''. *Journal of Financial Economics*5, 177-188.

Vasicek, O.A. and Fong, H.G. (1982): ''Term structure modeling using exponential splines''. *Journal of Finance* 37, 339-343.