MATHEMATICAL SURVEY OF THE TERM STRUCTURE OF INTEREST RATE: EVIDENCE FROM NIGERIAN EUROBOND

[1*]Ogungbenle Gbenga Michael and [2]Ogungbenle Simeon Kayode
gbengarising@gmail.com *kaykaska@yahoo.com*
[1]Department of Actuarial Science, University of Jos, Jos
[2]Department of Finance, Igbinedion University, Okada, Benin City

ABSTRACT
Term structure of interest rate describes the relationship between the yields on default-free securities which only differ in their term to maturity which mirrors the market future expectation for interest rates having consequences on fiscal policies with such factors such as interest rates and yield. The short-term yield $\alpha_0 + \alpha_1$ and the long-term yield α_0 react sharply to fresh market information which makes modeling and forecasting the term structure challenging. The paper aims to (i) determine how to predict the in-sample yield of time to maturity (ii) investigate how to use the parameters to determine the long-term and short-term yield. (iii) examine how the model fits into the observed data.(iv) determine the level, slope and curvature of Nigerian Eurobond. The statistical tools used for the analysis of data collected include ad-in Excel, SPSS 23 and the ordinary least square method. A linear regression was conducted and the results showed that the Nelson-Siegel model fits well into the observed data with R^2 adjusted of 90.8%.

Key words: Term–structure, Nelson-Siegel model, Yield curve, long term, short term

1. INTRODUCTION
Term Structure of Interest Rates describes the functional relationship between bonds of different terms, where the yield curve is plotted against their terms and interest rate provides useful information to the investing public. The objective of this paper is to predict the in-sample yield of time to maturity and then investigate how to use the estimated parameters to determine the long-term and short-term yield. This will hence enable us examine how well the model fits into the observed data so as compute the level, slope and curvature of Nigerian Eurobond. The curvature of the yield graph reflects the financial market future average for interest rates and conditions for fiscal policies. All debt instruments like bonds and derivative instruments are sensitive to interest rate scenerios. Furthermore interest rates are financial instruments employed in pricing all debt instruments since they are applied in time discounting. In a study conducted by Heath, Jarrow & Morton (1992), yield graphs plot the interest rates of some bond having homogeneous features against their corresponding different maturity terms, illustrating the relationship among yields on bonds that differ only in their terms to maturity.

The shape of the yield graph can demonstrate an upward sloping yield curve where the short term interest rates falls below the long term interest rates or may turn to be a downward sloping yield graph where the long term interest rates fall below the short term interest rates and is positively sloped or may be a flat sloping yield graph which occurs during which the short term interest rates are equal to the long term interest rates.

2 LITERATURE REVIEW

2.1 Theoretical Review

Following Pearson & Sun (1994), the term structure of interest rates defines the relationship between market interest rates on short term and long term securities and is usually observed as the difference in yield on fixed income securities due to the difference in the maturity of the instruments.

In Bakus (1997), the underlying hypothesis formulated to explain term structure of interest rates is the Expectations theory. According to the theory, long-term interest rates are estimated by the present level and by the expected trajectories of short-term interest rates and hence can be used to estimate future short-term interest rates. The initial model of the expectations theory maintains that the expected future short term interest rates implicit in the spot long-term interest rates are on average equal to the forward short-term interest rates. However, long-term interest rates may overestimate the future short term interest rates due to an additional return needed by an investor so as to choose investments in higher maturities. If this premium is constant, the expectations theory is valid but if the premium varies, the expectations theory is invalid. Inferences on the future trajectories of short-term interest rates could be drawn from the yield curve whenever the expectations theory remains valid. Since forward short-term interest rate serves as good indicators of sensitive macroeconomic variables, it is useful more or less to all market practitioners. Thus, the validity of the expectations theory permits financial institutions to examine the implications of the fiscal policies on the term structure of interest rates by ignoring the possibility of maturity segmentation in financial markets.

Following (Ho & Lee 1986; Elliot, Kwack & George 1986), term structure of interest rates defines the relationship among yields on debt instruments with similar tax, risk and liquidity features but resulting in distinct terms to maturity. The yield curve is constructed by plotting the interest rates of bonds against their terms. Here the term structure will then be the yield curve showing the functional relationship between spot rates of zero coupon securities and their term to maturity.

It is observed in (Vasicek, 1977; Aboagye, Akoena, Antwi-Asare & Gockel, 2008), interest rate is an economic price assigned to bring demand and supply of investible funds into equilibrium.Around 1986, the Nigerian interest rate was widely unstable before the introduction of the Structural Adjustment Program. The motive behind this decisive control of interest rates was motivated by different factors including the desire to smoothen the flow of credit to preferred sectors of the economy and the fact that market determined interest rates usually result into severe imperfection in lending rate which increases the cost of capital. An important consequence on the findings of Diebold & Li (2006) about better projection performance of the Nelson-Siegel model lies in forecasting the future yield accurately

Interest rate is usually set as a monetary policy by government to control fiscal variables such as inflation. The study of the term structure of interest rates relating to Nigerian Eurobond is thus necessitated by the fact that interest rates have a fundamental role to play in the development of Nigerian economy and also because of the vagaries of macroeconomic and microeconomic variables.

In Andersen & Lund (1997), we observe that Apex banks in the European Union apply appropriate model for modeling term structure where daily interest rate term structures is approximated by a predetermined model, however, in Nigeria there has not been any empirical evidence that the use of tested models have been deployed to examine term structure of interest rates.

From Balduzzi, Sanjiv, Silverio & Sundaram(1996), the term structure of interest rates represents the relation between interest rates or bond yields and different terms of maturities. Yields are observed to be related over time and over maturities and since short and long-term yields normally react sharply to fresh information, this makes modeling and forecasting the term structure a difficult job but despite this challenge, it remains a good tool due to its application in pricing market instrument. Earlier studies in Nigeria concentrated on either theoretical statistical analysis but in this paper our attention centers on more potent model by paying keen interest to the popular Dynamics of Nelson-Siegel model.

In a study conducted by Anyanwu (1995), it was found that like all other third world nations Nigeria has peculiar problem of determining the most appropriate fiscal policies which could be used to achieve economic growth but hence the process of the Nigerian economy requires correct scrutinization of the economic policies. Anyanwu (1995) observed some social-economic problems facing Nigeria to include pricing instability, high rate of unemployment index, balance of payment problems, debt crisis, political instability, capital fight, low capacity utilization, low savings, investment and income, downward trend in economic activities and lately economic depression. Many solutions have been suggested to solve these myriads of problems, both by individual and institutional and prominent among institutional solution IMF and World Bank whose advice resulted in the formulation of Structural Adjustment Program hypothesis in 1986.

2.2 **Empirical Review**

A number of studies have examined term structure of interest rate in other parts of the world.
Some of the studies carried out including Nigeria are stated as follows.

Poklepovic, Aljinovic and Marasovic (2013) calculated the yield curve of Croatian financial market for the period October, 2011-August, 2012, deploying Nelson-Siegel and Svensson models. It was reported that the estimation process based on Nelson-Siegel forcast producedpreferred result in the considering its smaller mean square error when compared with estimation based on Svensson model. However, Anyanwu and Oruh (2011) used non-parametric technique to generate the yield curve that may be used to price the Nigerian Treasury Bill with the help of stochastic differential equation tool box dynamics.

In Diebold & Li (2006), it was observed that the three factor Nelson-Siegel model can also be applied to construct correct term structure forecast. By applying a simple two-step estimation procedure, they reported that the model was well defined, relative to competing models, especially long forecast horizons. Monch (2006) partially confirmed these results while Fabozzi, Martellini and Prainlet (2005) proved that the Nelson-Siegel model produces projections that are both statistically accurate and financially meaningful as they can be applied to produce large investment returns.

3 MATERIAL AND METHODS: NELSON-SIEGEL TERM STRUCTURE OF INTEREST RATE

The parametric function-based models include the model suggested by
Nelson & Siegel (1987) and its extension in Svensson(1994). Nelson-Siegel (1985) formulated forward rate curve to be estimated as

$$g(\eta) = \alpha_0 + \alpha_1^{-\frac{\eta}{\tau}} + \frac{\eta}{\tau}\alpha_2^{-\frac{\eta}{\tau}} \qquad (1)$$

Seppala & Viertio (1996) reported that the above model has interesting economic implications for the parameters and good characteristics

$$\lim_{\eta\to\infty} g(\eta) = \alpha_{0,} and \lim_{\eta\to 0} g(\eta) = \alpha_0 + \alpha_1 \qquad (2)$$

The value of the parameter $\alpha_0 > 0$ represents the zero-coupon yield graph function.

The asymptote of forward curve is the long term contribution.The sum of Parameters $\alpha_0 + \alpha_1 > 0$ determines the initial value of the forward curve $g(0) = \alpha_0 + \alpha_1$ which can be interpreted as instantaneous spot rate hence $g(0) > 0$. The parameter α_1 is thus the deviation of the function values from the asymptote and defines the curvature of the function or the difference between long-term and short-term forward interest rates.

In a study carried out by Seppala & Viertio (1996), the Nelson-Siegel model approximates an arbitrage-free model where the authors first estimated the parameters using the Nelson-Siegel model and subsequently used the estimates to generate interest rate term structures as an input for arbitrage-free models. It was reported that parameters obtained from the
Nelson-Siegel model do not statistically differ from those obtained from the no arbitrage affine-term structure model. The Nelson-Siegel is highly non-linear and causes market experts to experience approximation problems. Nelson & Siegel (1987) however transformed the non-linear approximation problem into a simple linear problem, by fixing the shape parameter that causes the nonlinearity.

Nelson & Siegel (1987) specifies the forward rate curve $g(\tau)$ as follows.

$$g(\tau) = \begin{bmatrix} \alpha_0 \\ \alpha_1 \\ \alpha_2 \end{bmatrix} \begin{bmatrix} 1 \\ e^{-\frac{\tau}{\lambda}} \\ \dfrac{\tau e^{-\frac{\tau}{\lambda}}}{\lambda} \end{bmatrix} = \begin{bmatrix} \alpha_0 \\ \alpha_1 \\ \alpha_2 \end{bmatrix} \begin{bmatrix} u_0 \\ u_1 \\ u_2 \end{bmatrix} \qquad (3)$$

Where τ is time to maturity, α_0, α_1, α_2 and λ are coefficients with $\lambda > 0$.

u_0 is a constant, u_1 is an exponential decay function and a Laguerre function u_2 of the form ye^{-y} yielding the product of an exponential function with a polynomial. The constant represents the long term interest rate level. The exponential decay function reflects the second factor, a downward ($\alpha_1 > 0$) or an upward ($\alpha_1 < 0$) slope.

Nelson & Siegel (1987) used a first degree polynomial with implicit Laguerre function in the Nelson-Siegel model to generate a hump ($\alpha_2 > 0$) or a trough ($\alpha_2 < 0$). The bigger the absolute value of α_2, the wider the hump. The coefficient λ_t, referred to as the shape parameter, determines the both steepness of the slope factor and the location of the maximum Laguerre function. In a study conducted in, (Culbertson, 1957;Cox, Ingersoll & Ross, 1985; Schwartz, 1987; Buhler, Marliese, Ulrich, Thomas, 1999; Dai & Singleton, 2003), it was reported that the spot rate function is the average of the forward rate curve up to time to maturity τ and defined as: $u(\tau) = \frac{1}{\tau}\int_0^\tau g(s)ds$ (4)

This smooth function u can be numerically approximated as

$$u(\tau) = \frac{1}{\tau}\int_0^\tau g(s)ds = \frac{h}{3\tau}\left[g_0 + 4g_1 + 2g_2 + ... + g_n\right] - \frac{h}{90\tau}\left[\delta^4 g_1 + \delta^4 g_3 + ... + \delta^4 g_{n-1}\right] + \frac{h}{756\tau}\left[\delta^6 g_1 + \delta^6 g_3 + ... + \delta^6 g_{n-1}\right]$$

It is clear that with continuous compounding, the corresponding spot rate function at time to maturity τ is defined as:

$$u(\tau) = \begin{bmatrix}\alpha_0\\ \alpha_1\\ \alpha_2\end{bmatrix}\begin{bmatrix}1\\ \frac{\lambda(1-e^{-\frac{\tau}{\lambda}})}{\tau}\\ \frac{\lambda(1-e^{-\frac{\tau}{\lambda}})}{\tau} - e^{-\frac{\tau}{\lambda}}\end{bmatrix} = \begin{bmatrix}\alpha_0\\ \alpha_1\\ \alpha_2\end{bmatrix}\begin{bmatrix}u_0\\ u_1\\ u_2\end{bmatrix} \qquad (5)$$

$$\int_0^\tau g(s)ds = \tau * \begin{bmatrix}\alpha_0\\ \alpha_1\\ \alpha_2\end{bmatrix}\begin{bmatrix}1\\ \frac{\lambda(1-e^{-\frac{\tau}{\lambda}})}{\tau}\\ \frac{\lambda(1-e^{-\frac{\tau}{\lambda}})}{\tau} - e^{-\frac{\tau}{\lambda}}\end{bmatrix} \qquad (6)$$

The function $u(\tau)$ depicts the three building blocks of the Nelson-Siegel model. The curves u_0, u_1, u_2 represent the level, slope and curvature components of the forward rates (spot rates) curve. The original Nelson-Siegel model fits the yield graph with the simple functional form.

$$z(\tau) = \alpha_0 + \alpha_1\left(\frac{1-e^{-\lambda_\tau}}{\lambda_\tau}\right) + \alpha_2\left(\frac{1-e^{-\lambda_\tau}}{\lambda_\tau} - e^{-\lambda_\tau}\right) \qquad (7)$$

Where $z(\tau)$ is the zero-coupon yield with τ months to maturity and α_0, α_1, α_2 and λ are parameters. Diebol & Li (2006) suggested, the α coefficients to vary over time so that, given the Nelson-Siegel loadings, the coefficients could be described as time varying parameters, level slope and curvature factors. In order to describe this, the model above takes the form

$$z(\tau) = level_\tau + Slope_\tau (\frac{1-e^{-\lambda_\tau}}{\lambda_\tau}) + Curvature_\tau (\frac{1-e^{-\lambda_\tau}}{\lambda_{\phi\tau}} - e^{-\lambda_\tau}) \quad (8)$$

Nelson-Siegel Model is a powerful Term Structure forecasting method because it can generate minimal approximation of the yield graph with only few number of parameters. The three components

$$\left[1, \frac{1-\exp(-\frac{\tau}{\lambda_\tau})}{\frac{\tau}{\lambda_\tau}}, \frac{1-\exp(-\frac{\tau}{\lambda_\tau})}{\frac{\tau}{\lambda_\tau}} - \exp(-\frac{\tau}{\lambda_\tau}) \right] \quad (9)$$

allows enough flexibility for the model to include a band of monotonically humped and S-shapes structure usually observed in yield data. Moreover the model produces forward and yield graphs with the property of starting from an easily computed short rate value of α_1, α_2 and leveling off at a finite-maturity value of α_1 that is constant.

The three Nelson-Siegel components are clear explanation of short, medium and long term components, the nomenclature of which are the result of each element's contribution to the yield curvature. The long term component is the component on α_1 because it is constant at 1 and therefore the same for every maturity. The contribution of α_2 is

$$\left[\frac{1-\exp(-\frac{\tau}{\lambda_\tau})}{\frac{\tau}{\lambda_\tau}} \right] \quad (10)$$

and is described as the short-term component starting at 1 but exponentially decays to zero. The rate of decay is determined by the variable λ_t. Smaller values for λ_t result in a quicker decay to $o(1)$. The contribution of medium-term is

$$\left[\frac{1-\exp(-\frac{\tau}{\lambda_\tau})}{\frac{\tau}{\lambda_\tau}} - \exp(-\frac{\tau}{\lambda_\tau}) \right] \quad (11)$$

At zero, it increases for medium maturities and then decays to $o(1)$ again thereby creating a humped shape. The decay parameter λ_τ determines at which maturity this component attains its maximum.

4 DATA PRESENTATION AND ANALYSIS

The data used in the presentation of the model contains the daily closing of the Nigerian Eurobond from January to December 2018 to analyze and fit the Nigerian Eurobond yield curve using the Nelson-Siegel (1987) model. The data for the whole year (2018) was analyzed and the findings were also displayed in graphical and tabular form to include other statistic not captured on the curve and also to enhance easy access to the statistical values. The daily yields of the Nigerian Eurobond were extracted and analyzed based on descriptive analysis in order to have the necessary statistics for further analysis and also for an informed decision. For the purpose of this study, monthly data was analyzed and presented consecutively.

January 2018 contains only maturities (tenors) which are two years, four years, five years, nine years, fourteen years and twenty nine years with corresponding mean yield of 4.2177, 4.5687, 4.9120, 5.8516, 6.3551 and 6.9199. The short term in this month is two years and the long-term is twenty nine years. The four years maturity had the lowest volatility while the nine years tenor had the highest volatility. The yield curve movement was upward sloping from the first maturity to the last maturity. This indication verifies the premise that as time to maturity goes upward, the corresponding yield also moves in tandem to it.

The data of February extracted contains six tenors which are two years, four years, five years, nine years, fourteen years and twenty nine years with mean yield of 4.5771, 4.9511, 5.5252, 6.0651, 6.7428, and 7.3674 respectively. The month has a similar behavior as that of January. However, some little behavioral changes were observed such as: an increase in the overall yield of February compared to January, higher overall volatility with lower volatility on four years tenor and higher volatility at fifteen years tenor.

The month of March contains eight tenors with the introduction of twelve years tenor and twenty years tenor. All together we have tenors such as two years, four years, five years, nine years, twelve years, fourteen years, twenty years and twenty nine years with respective mean yield of 4.8155, 4.9715, 5.2553, 6.3986, 6.9005, 7.0586, 7.3835 and 7.4866, The yield curve here is upward sloping indicating the direct proportionality of yield against maturity.

April also presented eight tenors of two years, four years, five years, nine years, twelve years, fourteen years, twenty years and twenty nine years with average mean of 4.5139, 4.9520, 5.1594, 6.1730, 6.5987, 6.7417, 7.1384 and 7.2535 respectively. Yield curve slope remains upward. Low overall volatility can be observed across all the available tenors.

The month of May also presented eight tenors of two years, four years, five years, nine years, twelve years, fourteen years, twenty years and twenty nine years with respective mean yield of 5.0535, 5.2970, 5.6346, 6.6152, 7.1334, 7.2577, 7.6416 and 7.7094, Just as the previous month, the yield curve is also upward sloping given an increase in time to maturity.

The month of June has eight tenors of two years, four years, five years, nine years, twelve years, fourteen years, twenty years and twenty nine years with mean of 5.6589, 5.8075, 6.1907, 7.0967, 7.5357, 7.7163, 8.0087 and 8.1312 respectively. The month has the same behavioral characteristics as the previous month except that the overall variation of the observed data from the mean has increased.

The month of July also has eight tenors of two years, four years, five years, nine years, twelve years, fourteen years, twenty years and twenty nine years with respective mean of 5.1895, 5.6530, 6.1011, 7.1077, 7.4356, 7.6917, 7.9503 and 8.0993, Other characteristics apply as of the previous month.

August also contains eight tenors of two years, four years, five years, nine years, twelve years, fourteen years, twenty years and twenty nine years with respective mean of 5.0565, 5.6443, 6.1359, 7.3105, 7.6536, 7.9321, 8.2419 and 8.3637, The month has the same characteristics as the previous month.

The month of September has eight tenors of two years, four years, five years, nine years, twelve years, fourteen years, twenty years and twenty nine years with mean 5.2959, 5.8869, 6.3462, 7.4439, 7.8114, 7.9854, 8.3039 and 8.3921. The month yield curve is also upward sloping. Volatility is low when compared to the previous month.

The month of October also contains eight tenors which are two years, four years, five years, nine years, twelve years, fourteen years, twenty years and twenty nine years with respective mean of 4.9928, 5.6775, 6.1528, 7.3000, 7.7314, 7.8546, 8.2435 and 8.3326. The month has similar characteristics as the previous month.

November contains eight tenors which are two years, four years, five years, nine years, twelve years, fourteen years, twenty years and twenty nine years with respective mean of 5.2855, 5.9800, 6.5409, 7.7949, 8.2931, 8.4819, 8.7588 and 8.7440. The yield curve observed is upward sloping with a decline at the last tenor. Volatility is relatively low across all tenors presented.

The month of December contains the highest number of tenor of eleven tenors all together. This includes two years, four years, five years, seven years, nine years, twelve years, thirteen years, fourteen years, twenty years twenty nine years and thirty years with respective average yield of 5.9592, 6.4196, 7.2886, 8.1602, 8.3357, 8.6806, 9.0254, 8.9917, 8.9861, 8.9931 and 9.4185. The volatility in December is very low in comparison to the previous month. The yield curve observed here is upward sloping from the first tenor to the seventh tenor and then declines from the eighth tenor to the ninth tenor and then it continues to rise up to the eleventh tenor.

4.1 Overall descriptive statistics

We present the overall descriptive data to enable us analyze the aggregate model based as presented in the table below.

Table 1: Overall descriptive statistics

	N	Minimum	Maximum	Mean		Std. Deviation	Variance
Tenor ($\boldsymbol{\tau}$)	Statistic	Statistic	Statistic	Statistic	Std. Error	Statistic	Statistic
2 years	12	4.22	5.96	5.0513	.14021	.48569	.236
4 years	12	4.57	6.42	5.4841	.15527	.53788	.289

5 years	12	4.91	7.29	5.9369	.19301	.66861	.447
9 years	12	5.85	8.34	6.9577	.21647	.74988	.562
12 years	10	6.60	8.68	7.5774	.19575	.61901	.383
14 years	12	6.36	8.99	7.5675	.22155	.76746	.589
20 years	10	7.14	8.99	8.0657	.18162	.57432	.330
29 years	12	6.92	8.99	7.9827	.18324	.63475	.403
30 years	1	9.42	9.42	9.4185	.	.	.

Authors' computation via SPSS 23

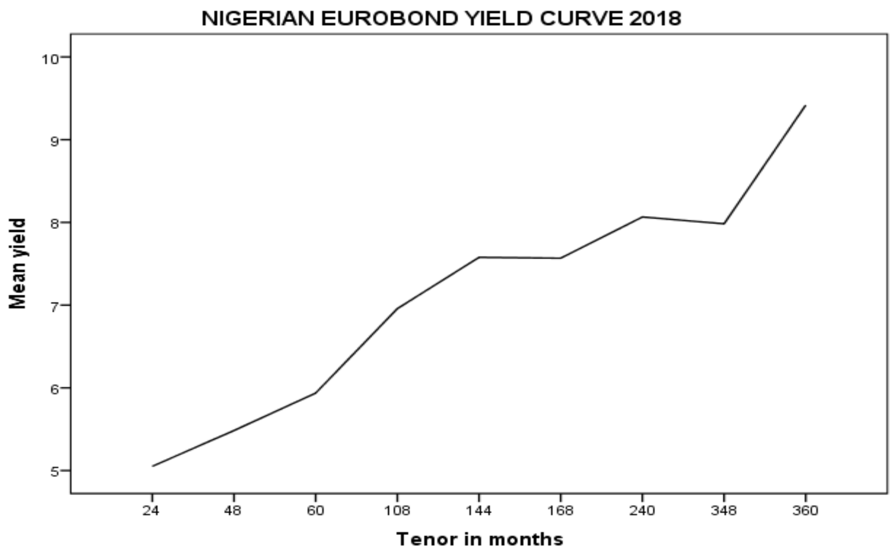

Figure 1. Authors' computation via spss 23

From the above table 1 the overall descriptive statistics presented nine tenors. This includes two years, four years, five years, nine years, twelve years, fourteen years, twenty years, twenty nine years and twenty nine years with respective average yield of 5.0513, 5.4841, 5.9369, 6.9577, 7.5774, 7.5675, 8.0657, 7.9027 and 9.4185. The data indicates a higher volatility over all tenors. The yield curve is shown above in figure 1. From the figure 1 above, we can observe that the yield curve is upward sloping that is, when time to maturity increases, the yields also increase. However, a little decline in the yield curve can be observed at fourteen years tenor and another decline in yield curve at the twenty nine years tenor. We can still conclude that the overall yield curve is upward sloping.

4.2 How do we predict the in-sample yield of τ?

As established earlier that the Nelson-Siegel can be used to estimate in-sample of τ (τ as time to maturity in month) given an observed yield, the model parameters was estimated using the ordinary least square method given the value of lambda as calculated previously.

The result is presented in table 2 below.

Table 2: Coefficients

Estimated parameters	Coefficients		Standardized Coefficients	T	Sig.
	α	Std. Error	Beta		
α_0	9.424	.373		25.249	.000
α_1	-4.027	1.400	-.579	-2.877	.028
α_2	-6.793	3.222	-.424	-2.108	.080

Authors computation via SPSS 23

From the above table 2, the parameters α_0, α_1 and α_2 are 9.424, -4.027 and -6.793 respectively. Substituting the parameters values into the model (7), we have:

$$z_t(\tau) = 9.424 - 4.027\left(\frac{1-e^{-0.03778438\tau}}{0.03778438\tau}\right) - 6.793\left(\frac{1-e^{-0.03778438\tau}}{0.03778438\tau} - e^{-0.03778438\tau}\right)$$

This means that, at any given value of τ the in sample yield can be estimated. However, since the in-sample yield does not exceed a maturity of more than 30 years, we write the conditional model for in-sample estimation of yield as

$$z_t(\tau) = 9.424 - 4.027\left(\frac{1-e^{-0.03778438\tau}}{0.03778438\tau}\right) - 6.793\left(\frac{1-e^{-0.03778438\tau}}{0.03778438\tau} - e^{-0.03778438\tau}\right) \qquad (12)$$

for $0 \le \tau \le 360$

4.3 How do we use the parameters to determine long-term yield and short-term yield?

The calculated parameters can be used to determine the short term yield and the long-term yield. As defined in Diebol and Li (2006), the first parameter α_0 is defined as long-term yield and ($\alpha_0+\alpha_1$) as short-term yield. Following the definition, the long-term yield is 9.424 which is close to the 30 years yield of 9.4185 with a difference of 0.0055. We can conclude that from the observed data, the long-term yield is α_0, The value of the defined short-term yield is 5.397 and the short-term of the observed is the two years tenor with the mean yield of 5.0513 with the difference of 0.3457. Though the difference between the observed and the defined yield is significantly large, however, it can be concluded that the defined short-term can be referred to as the short-term yield. The predicted yield curve and the observed yield curve are presented below.

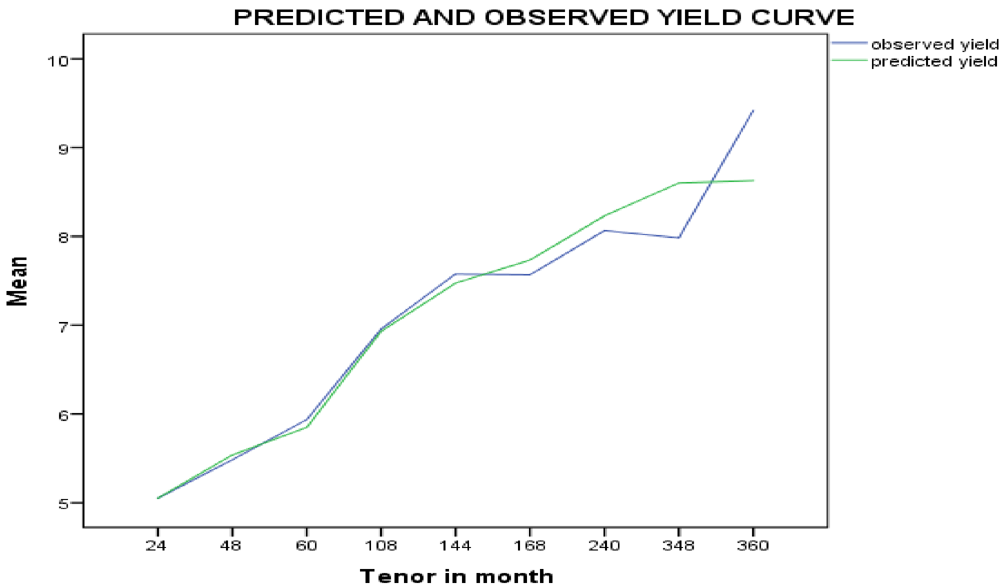

FIGURE 2. AUTHORS' COMPUTATION VIA SPSS 23

4.4 How does the model fit into the observed data?

The measured goodness of fit is determined when ordinary least square method is applied on data by R^2 adjusted. The model measure of fit analysis is depicted in table 3 below.

Table 3 **Model Summary**

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.965[a]	.931	.908	.425032156

Authors' computation via SPSS 23

The adjusted R square is the coefficient of multiple determinations which is the variance percentage in the dependent variable as explained by the independent variable.

From the above table 2, the R^2 is 0.931 and the R^2 adjusted is 0.908 with standard error of the estimate 0.425032156. This indicates that 90.8% (R^2 adjusted) of the observed data can be explained by the Nelson-Siegel model using the estimated parameters with 9.2% error. We can conclude that the Nelson-Siegel model in this study fits very well.

4.5 How can we estimate the level, slope and the curvature of the Nigerian Eurobond?

The Nelson–Siegel representation is interpreted as a dynamic latent factor model where α_0, α_1 and α_2 are time-varying parameters that captures the level, slope and the curvature respectively. From the above table 3 we define our level, slope and curvature as 9.424, -4.027, and -6.793

5 DISCUSSION OF RESULT

In the analysis of the data using the descriptive analysis, we found out that a higher percentage of the data analyzed is upward sloping. This has proved the premise that the yield of the Nigerian Eurobond is directly proportional to its time to maturity. This means that the higher the time to maturity, the higher the percentage yield. We also observed that the months with relatively higher yields across tenors, possessed higher level of volatility. This indicates that the higher the risk of the bond, the higher is the expected return, since variance is a measure of risk. The analysis of the Nelson-Siegel model used in this study shows that the in-sample of t (time to maturity) which are not included in the observed data, can be estimated at any point provided the value is not more than 360 months given the value of lambda and the estimated parameters. We also found that the estimated parameters of the Nelson-Siegel model analyzed, using the ordinary least square method, can be used to determine the long-term yield as well as the short-term yield. The findings shows that α_0 can be referred to as the contribution to long-term yield with 30 years maturity in this study while the short-term is determined by ($\alpha_0 + \alpha_1$) and in this study, 2 years is our short-term. The analysis of goodness of fit of the Nelson-Siegel model shows that the model fits in well into the observed data. This is indicated by R square adjusted with the value as 0.908 which means that 90.8% of the observed data can be explained by the model.

CONCLUSION:

The study examines the evolution between the yield curve and the Nigerian economy with a particular attention to Nelson-Siegel model. The yield curve model of this study implicitly considers the yield factors such as level, slope and curvature to construct our model based on Nelson-Siegel to distill the term structure of interest rate. A sensitive indicator of the term structure of interest rate, is the forward rate curve responsible for the prediction of expected future time trajectories of future short term rates

Based on the data obtained in this paper, our findings show the importance of the yield graph on the Nigerian Eurobond using the Nelson-Siegel model. Furthermore, we also found out how useful the yield graph is to operators of financial markets since the shape of the yield graph reflects the market future average for interest rates and fiscal policies. The pivot of the term structure of interest rates in this paper using the Nelson-Siegel model is the λ value which we solved using non-constrained optimization techniques as seen in equation (12). Based on our results, we suggest the following policy recommendations.

(1). Accurate data for the Nigerian Eurobond should be carefully analyzed by financial experts so as to mitigate errors to enable researchers obtain data used in reporting in-sample and out of sample statistics efficiently. (2). Efficient models should be deployed to help analyst predict the yield graphs on a daily basis because if the term structure model used is too flexible, unwanted measurement errors may occur in the model, since the estimated term structure depends on the estimation method used. (3). Investors should pay attention to the underlying context whenever straight line(flat) yield graph is produced where the difference in yield between short-term and long-term bonds may not be favourable since investing in long-term bonds will be preferred over short-term bond as the yields on short-term bonds will be higher leading to lower prices. (4). Financial institutions should involve experts to help in yield forecasting methodologies using the yield graph since the yield graph is a description of what it costs to borrow funds at various time.

REFERENCES

Aboagye A. Q. Q, Akoena S .K, Antwi-Asare T. O., Gockel A. F, (2008), ‘’Explaining interest rate spreads in Ghana’’. *African development Review* vol., 20, pp. 378-399.

Andersen T. G, Lund J, (1997), ‘’Estimating continuous-time stochastic volatility models of the short-term interest rates’’. *Journal of Econometrics* vol. 77(8), pp. 343-377.

Anyanwu M.C and Oruh B.I (2011), ‘’ Modelling the term structure of Nigeria Treasury Bill interest rate with Cox, Ingersol, and Ross Model. *Journal of computer and mathematical sciences* vol. *2(5),* pp. *693-799*

Anyanwu, J. C, (1995), ‘’Structural adjustment programme in Nigeria and its implication’’ *Socio-economic development Journal*. vol. 6(2), pp. 21-30

Bakus D. (1997), ‘’Expectations Theory’’. *Journal of Banking and Commerce.* vol. 2(9), pp. 1204-5357.

Balduzzi P, Sanjiv R, Silverio F, Sundaram R(1996), ‘’A simple approach to three factor affine term structure models’’. *Journal of Fixed Income* vol. 6(9), pp. 43-53.

Buhler, W. Marliese, U, Ulrich, W. Thomas, W. S, (1999), ‘’An empirical comparison of forward rate and sport rate models for valuing interest rate options’’. *Journal of Finance.* vol4(8), pp. 23-29

Cox, J. Ingersoll. J and Ross, S. (1985), ‘’A theory of term structure of interest rate’’. *Econometrica,* vol. 53, pp. 385-408.

Culbertson J. M, (1957), ‘’The term structure of interest rates’’. *Quarterly Journal of Economics* vol. 71(4), pp. 485-517.

Dai Q, Singleton K, (2003), ‘’Term structure dynamics in theory and reality’’. *Review ofFinancial Studies* 16, 631-678.

Diebold, F. X, & Li. C, (2006), ‘’Forecasting the term structure of Government bond yields’’, *Journal of Econometrics,* vol. 130(1), pp. 337-364

Elliot J, Kwack S. Y, George S. T (1986), ‘’An Econometric model for the Kenyan economy’’. *Econometric Modeling* vol. 3, pp. 12-30.

Fabozzi, F.J., Martellini, L., and Praiulet, P (2005), ‘’Predictability in the shape of the terms structure of interest rates’’. *Journal of fixed Income,* vol. 59, pp. 40-53.

Heath D, Jarrow R, and Morton A, (1992), ''Bond pricing and term structure of interest rates: A new methodology for contingent claims valuation''. *Econometrica* vol. 60, pp. 77-105.

Ho, T. S. Y and Lee, S. B (1986), ''Term structure movement and pricing interest rates contingent claims''. *The journal of finance*, vol. 41(5), pp. 1011-1029.

Monch E, (2006), *Forcasting the yield curve in a data-rich environment, a non-arbitrage factor*. Working Paper, Humboldt University, Berlin

Nelson C and A Siegel (1987), ''Parsimonious modelling of yield curves'' *Journal of Business*. Vol. 60-4, pp. 473-489.

Nelson, C. R. and Siegel A. F (1985), ''Parsimonious modelling of yield curves for US Treasury bills''. *The journal of business* vol. 60-4, pp. 473-489

Pearson, N. D and Sun, T (1994), ''Exploiting the conditional density in estimating the term structure. An application to the Cox. Ingersoll and Ross model''. *The Journal of finance*. vol. 48(5), pp. 79-1304.

Poklepovic T, Aljinovic Z, and Marasovic B,(2014), The ability of forecasting the term structure of Interest rates based on Nelson-Siegel and Svensson Model'' . *International Journal of Economics and Management Engineering*, vol. 8(3), pp 718-724

Schwartz,(1987), ''A two-factor model of the Term Structure: An Approximation Analytical Solution''. *Journal of financial and Quantitative Analysis* vol. 19, pp. 413-424.

Seppala and Vertio (1996), ''Interpretation of parameters and good asymptotical characteristics'', *European, Financial and accounting Journal*, vol. 7, pp. 36-55

Svensson, L .E, (1994), *Estimating and interpreting forward interest rates: Sweden 1992-1994* [on-line], Washington, D. C. National Bureau of Economic Research working paper no 4871.

Vasicek, O.A. (1977), ''An equilibrium characterization of the term structure''. *Journal of Financial Economics* vol. 5(4), pp. 177-188